

ASSESSMENT OF THE SRSI METHOD USING THE THERMAL RADIATION
OF WATER-VAPOR LAYER

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The method of "sums of rectangles for spectral intervals" is assessed by the absorption and radiation of energy by a water-vapor layer with a linear temperature field.

Introduction. Heat exchange by radiation depends essentially on the spectra of bodies. In ovens and furnaces and lining surfaces of the bodies being heated, and suspended phases have continuous spectra while the gaseous components of the media have molecular spectra. The difficulties involved in correctly taking them into account simultaneously have not yet been overcome. Elsewhere [1] we proposed the approximated method of "sums of rectangles for spectral intervals" (SRSI). The scale of the spectra is divided into intervals with fixed boundaries. The optical constants of bodies with a continuous spectrum are averaged inside the intervals. In the same intervals molecular spectra are described by rectangles with heights corresponding to the absorption coefficients and widths equal to the energy fractions. In the limit of narrowing of the intervals the absorption coefficients are equalized to a monochromatic coefficient. In the other limit, i.e., expansion of the intervals and reduction of their number to one with a complete spectrum, with simplification of the weighting factors, the SRSI method degenerates into the Hottle "method of a suspended sum of gray gases."

The SRSI method can be used on satisfaction of some assumption. Its error is small when the complete spectrum is divided into only nine intervals. In [1] we obtained control estimates for a water-vapor layer under isothermal conditions. Incidentally concepts were expressed after the Method of V. G. Lisienko and Yu. A. Zhuravlev (LZh).

1. Absorptivity of Water Vapor. The central quantity used in the SRSI method is the directed absorptivity of a gas of depth x with a mean temperature \bar{T} at this depth for the incident flux with a blackbody spectrum at T_0 ,

$$\tilde{a}_0 = \sum_{i=1}^L g_i \sum_{m=1}^M c_{lm}^0 [1 - \exp(-K_{lm}x)]; \quad g_i = \sum_{j=1}^{N_i} \pi \Delta\omega_j I_{0j} / \sigma T_0^4, \quad (1)$$

where $c_{lm}^0 \equiv c_{lm}^0(T_0; \bar{T})$, is the weight of the m -th rectangular band in the l -th interval of the spectrum. Under isothermal conditions $\bar{T} = T$. As $T_0 \rightarrow T$, $c_{lm}^0 \rightarrow c_{lm}(T)$ Eq. (1) gives the degree of blackness of the gas, $\tilde{a}_0 \rightarrow \tilde{\epsilon}$.

In accordance with (1) the system of heat-exchange equations is solved $L(M + 1) + L_0$ times over all parts of the spectrum. The flux densities obtained are added together. For the $M + 1$ segment inside the l -th interval $K_{lm} = 0$. It combines transparency microwindows. The number L_0 takes into account transparency macrowindows between bands and beyond the limits of the active spectrum of the gas.

The known studies were compared primarily by the initial base data. Our solutions for water vapor, up to 1988, were based on the absorptivity formula [2], matched with the data of Hottle. Starting from [3], we have used a new base of spectral data after [4]. Table 1 compares the values of a_0 . The agreement is good at $\bar{T} = 1000$ K, but as T increases the new values grow increasingly. Hottle's data are based on very limited observations. The new data are reliable and can be used for extrapolation with respect to the optical thickness and effective pressure.

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TABLE 1. Directed Water Vapor Absorptivity at \bar{T} for Incident Flux with Blackbody Spectrum at T_0 ; $P = 1$ atm; $p = 0$ atm (with infinite dilution with nitrogen). The figures after the decimal point are shown: upper, according to the previous base of initial data [2]; lower, according to the new base [4]

\bar{T} , K	T_0 , K	x , cm·atm					
		3	30	50	100	300	1000
1000	1500	50	169	202	250	337	425
		47	157	194	252	357	482
	2000	36	121	144	177	234	288
		32	112	140	185	269	376
	3000	27	93	110	134	169	198
		16	60	76	102	154	224
1500	1000	53	237	293	375	519	689
		76	279	343	438	592	748
	2000	27	122	151	192	260	331
		31	138	177	241	362	513
	3000	20	93	116	148	194	236
		16	74	97	137	215	323
2000	1000	43	228	289	374	525	707
		77	330	411	526	704	866
	1500	31	119	208	271	377	501
		43	158	291	394	577	775
	3000	16	90	117	154	209	260
		14	84	116	170	287	446

2. Averaging the Temperature on the Absorption Portion. The absorptivity, in contrast to the degree of blackness, remains meaningful with any temperature field. The problem of averaging the temperature over the absorption part x arises in the SRSI method. A simpler approximation, which was given preference in the end, was proposed by A. S. Nevskii:

$$\bar{T}_1 = \int_0^1 T dv, \text{ where } v = x'/x, x \geq x' \geq 0.$$

In [5] the temperature was averaged on the basis of a three-parameter approximation in absorption calculations for a gas with a vibration-rotation spectrum:

$$\bar{T}_2^m = \int_0^1 T^{2m-1} dv / \int_0^1 T^{m-1} dv,$$

where m depends on the effect of one parameter or another of the spectral band.

Basically, we must take $m = 0.5$, $1/\sqrt{\bar{T}_2} = \int_0^1 dv / \sqrt{T}$.

Some researchers assign an exclusive role to the gas density when conducting the calculations [6]. Then

$$\bar{T}_3^{-1} = \int_0^1 T^{-1} dv.$$

With any temperature field $\bar{T}_1 \geq \bar{T}_2 \geq \bar{T}_3$ but \bar{T}_2 is closer to \bar{T}_3 .

These data of the approximation are checked in as pure a form as possible on the example of the directed absorptivity of a water-vapor layer at $p = 0.1$ atm and $P = 1$ atm in a mixture with nitrogen with a linear temperature field. The standard of comparison was obtained from the bulk parameters k_j and a_j of the model of a narrow spectral band, which were adopted in [4] and were averaged in the Curtis-Godson (CG) approximation:

$$a_0 = \frac{\pi \Delta \omega}{\sigma T_0^4} \sum_{j=1}^{439} I_{0j}(T_0) \varepsilon_j; \varepsilon_j = 1 - \exp\left(-\frac{\bar{k}_j x}{\sqrt{1 + \bar{k}_j x / a_j}}\right), \quad (2)$$

TABLE 2. Discrepancy $(1 - \bar{a}_0/a_0) \cdot 100\%$ of the Approximate Directed Absorptivity of a Water-Vapor Layer of Thickness x_0 , cm·atm, with Linear Temperature from 1300 to 2600 K.

The value of \bar{a}_0 was calculated with $\bar{T}_1 = \int_0^1 T dv$ from the same formula (2) as used for a_0 .

T, K	Relative error = $(1 - \text{CG approx.}) \cdot 100\%$							
	$x_0=3$	30	50	100	300	1000	3000	10,000
600	3,55	3,98	3,64	2,92	1,76	1,28	0,85	0,18
1000	3,60	3,37	3,15	2,83	2,58	2,52	1,69	0,38
1500	3,65	3,02	2,96	3,01	3,29	3,21	2,12	0,64
2000	3,69	2,98	3,03	3,27	3,72	3,49	2,30	1,02
2500	3,71	3,02	3,13	3,46	3,95	3,58	2,38	1,43
3000	3,72	3,06	3,20	3,58	4,07	3,59	2,44	1,81

TABLE 3. Density of Fluxes q_- and q_+ at the Boundaries of a Water-Vapor Layer. The upper numbers are from [10] and the lower numbers, from the present standard with a new base of spectral data: $T_1 = 1000$ K and $T_2 = 2000$ K, $P = 1$ atm, $p = 0$

x_0 , cm·atm	q_- , kW/m ²	q_+ , kW/m ²	x_0 , cm·atm	q_- , kW/m ²	q_+ , kW/m ²
0,1	0,824	0,865	50	30,5	39,1
	0,671	0,676		52,9	115
1	6,84	7,79	100	67,9	107
	5,70	6,04		62,4	151
5	21,6	29,6	300	88,2	154
	19,7	23,4		—	—
10	30,8	48,1		122	250

where

$$\bar{k}_j = \int_0^1 k_j dv; \quad \bar{k}_j \bar{a}_j = \int_0^1 k_j a_j dv. \quad (3)$$

Comparison of the approximate and correct values \bar{a}_0 and a_0 from (1) and (2) gives the total error of the SRSI method. The error in finding the mean \bar{T} , however, must be found here in a more or less pure form. The approximate values of \bar{a}_0 contained in Table 2 were obtained from the same formula (2), but with k_j and a_j at temperature \bar{T} . We interpolated k_j and a_j from their tabulated values over the entire temperature field from 300 to 3000 K by the cubic spline method [7]. Table 2 includes discrepancies only for the simplest averaging $\bar{T}_1 = \int_0^1 T dv$ since they were the smallest.

Part of the error (see Table 2) was due to the CG approximation, i. e., the standard values of a_0 are not entirely correct. The part of the error due to the CG approximation is less than 2%, but may be substantially larger. A more correct standard of comparison can be obtained in principle from the "line-to-line" algorithm. Thus far, it has shown to be complete and reliable for carbon monoxide [8]. For water vapor the range of application of the algorithm is limited to part of the spectrum [9] and has an error that increases with the temperature; the algorithm is not used above 1500 K. We consider \bar{T}_1 to be the best approximation.

3. Radiation of a Single Layer of Gas with an Arbitrary Temperature Field. When \bar{T} has been chosen the SRSI method can be used for any temperature field. We consider the same kind of radiation as before, that of a layer of water vapor, but only a single layer. From a diagram (see Fig. 1) we determine the densities of the intrinsic thermal fluxes q_- and q_+ at the boundaries. This problem has long been a standard for comparing methods of calculation [10]. This is the first time that a new data base has been adopted. The standard of comparison here, too, was obtained in the CG approximation. In the one-dimensional case

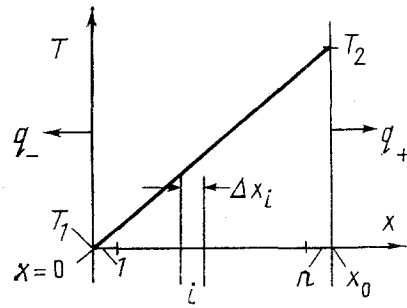


Fig. 1. Diagram for calculation of the self-radiation of water-vapor layer; q_- and q_+ (kW/m^2) are the fluxes at the "cold" and "hot" boundaries.

TABLE 4. Density of Fluxes q_- and q_+ at the Boundaries of a Water-Vapor Layer. The upper numbers were obtained by the SRSI method and the lower numbers, from the standard of comparison. A new base of spectral data; $T_1 = 1300$ K and $T_2 = 2600$ K, $P = 1$ atm, $p = 0$

x_0 , cm·atm	q_- , kW/m ²	q_+ , kW/m ²	x_0 , cm·atm	q_- , kW/m ²	q_+ , kW/m ²
0,1	1,02	1,05	100	233	347
	1,04	1,05		238	350
1	9,21	9,82	300	336	618
	9,68	9,91		357	615
5	38,0	42,3	1000	—	—
	38,9	42,3		464	1008
10	65,0	75,5	10 000	—	—
	65,3	74,6		431	1724
50	164	230	100 000	—	—
	172	233		286	2080

$$q_- = \pi \sum_{i=1}^n \sum_{j=1}^{439} I_{0j}(T_{i-1/2}) \Delta\omega_j [\epsilon_{ji}(T_{i-1/2}; x_i) - \epsilon_{j,i-1}(T_{i-1/2}; x_{i-1})], \quad (4)$$

where $T_{i-1/2}$, the temperature averaged over the i -th zone, is determined from the center of the zone;

$$T_{i-1/2} = T_1 + (T_2 - T_1)(i - 0,5)/n. \quad (5)$$

The parameters \bar{k}_j and \bar{a}_j in ϵ_j are averaged over the segments x_i and x_{i-1} , where x_i is the distance from the furthest boundary from the edge of the layer and $x_{i-1}|_{i=1} = 0$.

The value of q_+ is calculated from the same formula but with the function

$$T_{i-1/2} = T_2 - (T_2 - T_1)(i - 0,5)/n. \quad (5a)$$

By the SRSI method

$$\tilde{q}_- = \sum_{i=1}^L \sum_{i=1}^n \sigma T_{i-1/2}^4 g_i(T_{i-1/2}) [\tilde{a}_{0i}(x_i) - \tilde{a}_{0i}(x_{i-1})],$$

where

$$\tilde{a}_{0i}(x_i) = \sum_{m=1}^M c_{im}^0 [T_{i-1/2}; \bar{T}(x_i)] [1 - \exp(-K_{im}x_i)].$$

The flux density q_+ is calculated from the same formula with the function $T_{i-1/2}$ from (5a). The details of the SRSI method were published in [1].

TABLE 5. Density of Fluxes \bar{q}_- and \bar{q}_+ at the Boundaries of a Water-Vapor Layer by the LZh Method (version 2); $T_1 = 1300$ K, $T_2 = 2600$ K, $P = 1$ atm, $p = 0$; $\Delta = (1 - q/\bar{q})100\%$; q are the standard values from Table 4

x_0 , cm·atm	$n=20$				$n=5$			
	\bar{q}_- , kW/m ²	Δ_- , %	\bar{q}_+ , kW/m ²	Δ_+ , %	\bar{q}_- , kW/m ²	Δ_- , %	\bar{q}_+ , kW/m ²	Δ_+ , %
0,1	1,10	5,6	1,07	2,7	1,14	9,7	1,03	-1,10
1	10,7	10,3	10,5	6,1	10,9	13	10,0	1,2
5	47,7	21,8	48,2	14	46,7	20	45,1	6,6
10	84	28	88	18	81	24	82	9,7
50	241	40	300	29	226	31	276	19
100	325	37	461	32	308	29	433	24
300	412	15	793	29	427	19	821	33

Table 3 compares the best results from [10] with the present standard at $T_1 = 1000$ K and $T_2 = 2000$ K. The discrepancies, as in Table 1, are due mainly to the transition to a new base of spectral data. Subsequent calculations were carried out on the new base with a higher temperature field, $T_1 = 1300$ K and $T_2 = 2600$ K. Table 4 compares the SRSI method with the standard solution. The coefficients $c_{\ell m}$ and $K_{\ell m}$ were determined in the range of arguments $300 \geq x_0 \geq 0.1$ cm·atm, $2600 \geq T \geq 1000$ K. We assumed that: $L = 8$ with the boundaries of the intervals ℓ fixed in accordance with [1], three gray gases and one pure gas ($M = 3$), four terms of the polynomials for $c_{\ell m}$, and $T = \bar{T}_1$. The SRSI results give good agreement with the standard of comparison.

The SRSI method does not have extrapolation capabilities with respect to the arguments, whereby the value of x_0 is limited to 300 cm·atm in the calculations. The standard, conversely, does permit a growth of x_0 . The increase of x_0 to 10^5 cm·atm in Table 4 may be considerably inadmissible, but the results correspond to the general concepts. Water vapor becomes grayer as x_0 increases. The function $q_-(x_0)$ passes through a maximum, as for a gray gas. Becoming denser, the "cold" boundary zones shield the "hot" radiation of the deeper zones and q_- decreases from certain values of x_0 . This comment does not pertain to $q_+(x_0)$.

4. Imitation of the Method of V. G. Lisienko and Yu. A. Zhuravlev. The method cannot be imitated fully because of the differences between the bases of initial data. All the deviations, however, are treated in the verification of the method. For example, our studies showed that averaging the Planck function over such wide intervals of the spectrum and taking it out of the integral over the spectrum introduce a large error. The use of the energy fraction g_ℓ of the interval removes this difficulty. The number of intervals is the same in the LZh method and in the method we propose. We did take different interval boundaries [1], but that is unimportant. The verification of the LZh method was limited to isothermal conditions in [1]. The comparative calculation below is carried out for a linear temperature field.

In accordance with the LZh method the medium inside the ℓ -th interval is assumed to be gray with a local absorption coefficient

$$\bar{K}_\ell = \Delta S_\ell / \Delta \omega_\ell = \sum_{j=1}^{N_\ell} k_j / N_\ell, \quad (6)$$

where ΔS_ℓ is the sum of the integrated intensities of bands and lines in the ℓ -th interval of width $\Delta \omega_\ell$, $\text{cm}^{-1}(\text{cm}\cdot\text{atm})$.

The directed degree of blackness of the volume zone in the complete spectrum is calculated in the first approximation

$$\tilde{\epsilon} = \sum_{i=1}^L g_i [1 - \exp(-\bar{K}_i \Delta x)] = \sum_{i=1}^L g_i \tilde{\epsilon}_i,$$

where Δx is the zone size, cm·atm.

Then all the values of $\tilde{\epsilon}_\ell$ for the intervals of the spectrum are correlated by the factor $\epsilon/\tilde{\epsilon}$ where ϵ and $\tilde{\epsilon}$ are the integrated degrees of blackness, is also correct in the first approximation

$$\tilde{\varepsilon}_{l1} = \tilde{\varepsilon}_l(\varepsilon/\tilde{\varepsilon}) = 1 - \exp(-K_{l1}\Delta x).$$

The calculated value of the local absorption coefficient (zonal coefficient) is obtained from

$$K_{l1} = -\ln(1 - \tilde{\varepsilon}_{l1})/\Delta x. \quad (7)$$

The sum of the interval values $\tilde{\varepsilon}_{l1}$ with appropriate weights gives the correct value of the degree of blackness in the complete spectrum:

$$\sum_{l=1}^L q_l \tilde{\varepsilon}_{l1} = \varepsilon.$$

But this does not eliminate the errors upon the introduction of a gray local absorption coefficient K_{ℓ} . Coefficient K_{ℓ} from (6) has Bouger properties, i.e., depends only on local parameters of state of the medium. The correction coefficient $K_{\ell 1}$ from (7) also depends on the zone size Δx . We omit the results of comparison in the given version of the LZh method because the error was large but we do give them in the second version with a smaller error, using the correct value of the degree of blackness in the ℓ -th interval:

$$\varepsilon_{l2} = \frac{\sum_{j=1}^{N_l} I_{0j} \varepsilon_j}{\sum_{j=1}^{N_l} I_{0j}}.$$

On substituting ε_{l2} into (7) we find $K_{\ell 2}$. In the general case Δx is determined from the effective zone thickness. In the given one-dimensional case this is the zone size along the x-axis.

The density of the intrinsic flux at the boundary of a layer with reduced thickness x_0 , cm·atm, is summed over all zones:

$$q_- = \frac{x_0}{n} \sum_{i=1}^n \sigma T_{i-1/2}^4 \sum_{l=1}^L g_{l,i-1/2} K_{li} \exp\left(-\frac{x_0}{n} \sum_{i'=1}^{i-1/2} K_{li'}\right).$$

The index $i-1/2$ indicates that the parameters are calculated with the temperatures at the midpoints of the zones:

$$g_{l,i-1/2} = \frac{\pi \Delta \omega}{\sigma T_{i-1/2}^4} \sum_{j=1}^{N_l} I_{0j}(T_{i-1/2}).$$

In the calculations of q_- and q_+ the temperature is calculated from (5) and (5a). As before, $T_1 = 1300$ K, $T_2 = 2600$ K, $P = 1$ atm, $p = 0.1$ atm; the number of zones varies: $n = 5, 10,$ and 20 . The results from version 2 are given in Table 5. The LZh gives rise to the problem of choosing the zone size. Even at $n = 5$ the sizes were underestimated. The values obtained for $K_{\ell 2}$ from (7) were overestimated. Accordingly, the fluxes are also overestimated. Only one value at $x_0 = 0.1$ cm·atm with a negative error is the exception. The results are paradoxical: an increase in the number of zones resulted in a larger error. Experience from using narrow-band models of the spectrum shows that replacement of the molecular spectrum with a continuous spectrum distorts the results even in very narrow intervals, $\Delta \omega = 5$ cm⁻¹. In the given calculation the intervals are expanded by two or more orders of magnitude. The medium inside them was assumed to be gray. The SRSI method is distinguished by the fact that the molecular spectrum inside a wide interval is reconstructed into a sum of rectangles. The reconstruction is accomplished by methods of applied mathematics, but can also be done graphically.

NOTATION

a_0 , directed absorptivity of the medium at temperature \bar{T} for the incident flux with a blackbody spectrum at temperature T_0 ; $c_{\ell m}$, the energy weight of the m -th rectangle of the ℓ -th interval of the spectrum, dimensionless; g_{ℓ} , energy fraction of the ℓ -th interval with respect to the blackbody spectrum; k_j and a_j , the parameters of the model in the j -th elementary band of the spectrum, (cm·atm)⁻¹ and dimensionless; p and P , partial and total pressure, atm; q , flux density, W/m²; x , reduced depth, cm·atm; $K_{\ell m}$, absorption coefficient for the m -th rectangle of the ℓ -th interval of the spectrum (cm·atm)⁻¹; K_{ℓ} , average absorption coefficient in the ℓ -th interval from the continuous (gray) spectrum, (cm·atm)⁻¹; T and T_0 , temperatures of the medium and surfaces, K; N_{ℓ} , number of elementary intervals in the ℓ -th wide interval I_{0j} , Planck intensity, cm·W/(m²·sr); ε , degree of blackness; and ω , wave number, cm⁻¹. Indices: ℓ , wide inter-

val; L, number of intervals; m, small rectangle of spectrum; M, number of rectangles with $K_{\Omega m} > 0$; and \sim , approximate value.

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EFFECT OF THE PHYSICAL PROPERTIES OF A MATERIAL ON THE ELECTRODE EROSION IN A HIGH-CURRENT PULSED DISCHARGE

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Correlation analysis of the experimental data on the erosion of different metals for pulsed currents ranging from 2 A to 800 kA has determined that the erosion depends on the physical properties of the electrode material. The nature of the given relation is found to be determined by the amplitude of the discharge current and the electrode separation. Conclusions are made about the mechanisms of electric erosion of electrodes.

Introduction. The electric erosion of electrodes in a pulsed discharge is seriously hindered by the unsteady nature of the erosion processes, the fact that the current density has not been established, etc. (see [1-4]). Experimental determination and subsequent theoretical analysis of the dependence of erosion on the physical properties of the electrode material are one way of obviating these difficulties to a considerable degree. In this domain it is necessary to point out studies for a vacuum arc with a current of 80-800 A [3] as well as for a pulsed gas discharge with a current of 250 A [5] and 150-800 kA [6]. At the same time virtually no use is made of the results of experiments [7-24], in which the erosion of different metals was compared over a wide range of energy conditions, but as a rule the data were not processed mathematically and were not compared with each other.

In the present study, by means of mathematical processing and systematization of the results of [5-24] we find the dependence of erosion on the physical properties of a metal for pulsed currents of 2 to 800 kA.

Dependence of Erosion on the Physical Properties of the Electrode Material. The mathematical processing of the experimental data consisted of using correlation analysis, i.e., in each series of erosion resistance for each physical parameter the well known procedure [25] is used to calculate, "clean up," and evaluate the statistical significance of the correlation coefficients characterizing the coupling constant between the erosion and the

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